Learning from Order Examples

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Learning from Order Examples (LOE)

is the learning task that handles order specifications

- Formalize the LOE
- Propose several naive solution methods
- Experiments on Artificial Data:

  Analyze the charactristics of these methods
- Experiments on Real Data:

  Analyze the availability on Real Data

What is the "order" ?

Order: sorted lists of items according to some criterion

ex: an sorted list of sushi types (items) according to my preference (a criterion)

fatty tunas > eggs > squids

This order specify that

"I prefer fatty tunas to eggs"

This order does NOT specify that

"How much I prefer fatty tunas to eggs"

※ "sushi" is a kind of Japanese food

Merit for Using Orders

An Application to a sensory test

SD (Semantic Differential) method

ex: specify one's preference by the following scale

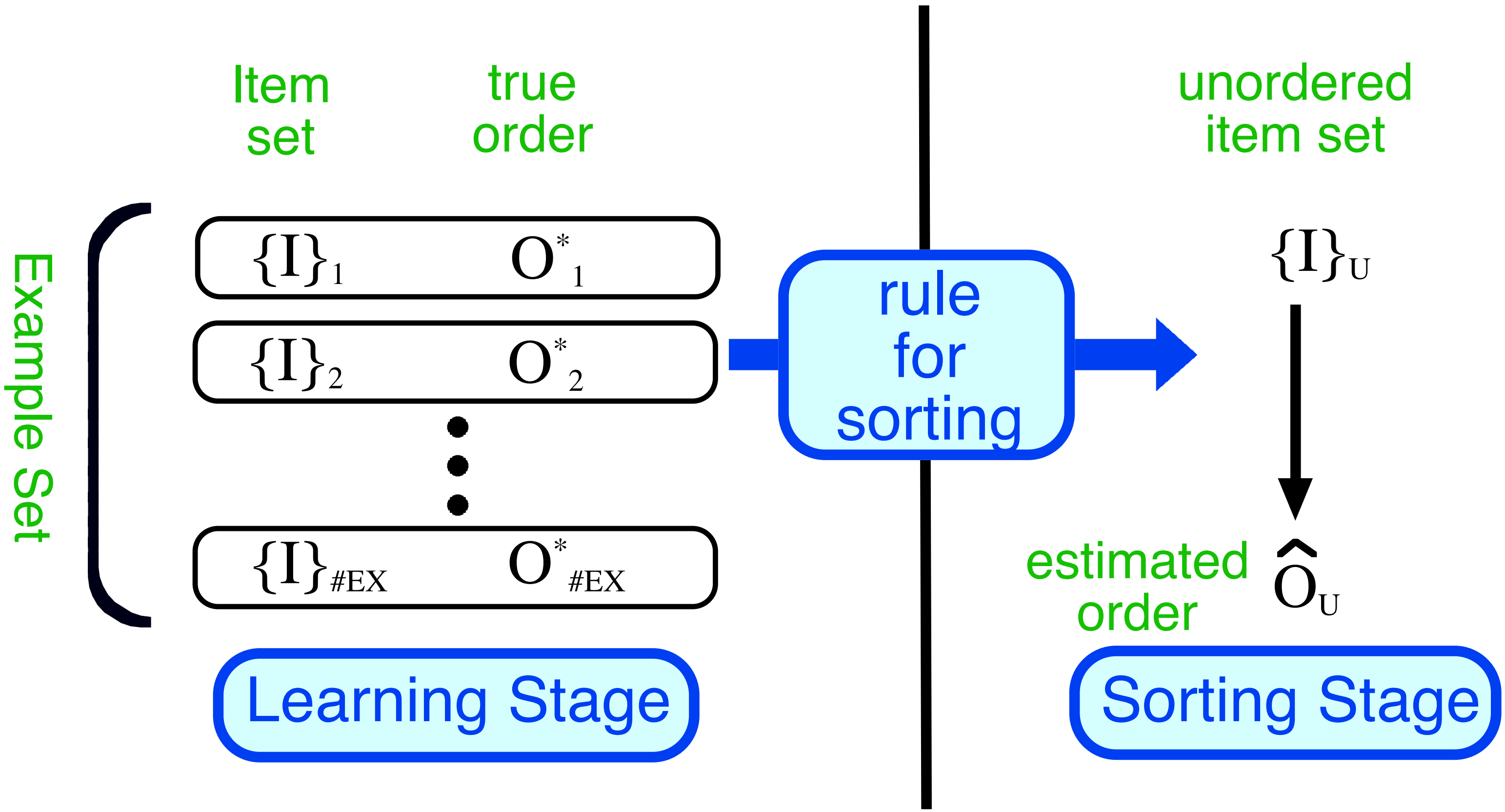
[like] 5 4 3 2 1 [dislike]

assumption: all respondents share an understanding of its range, divisions and extremes.

➡ **UNREALISTIC ASSUMPTION**

Specifications by using orders does not demand such an assumption

Formalization of LOE



Items are described by attribute value vectors

➡ The acquired rule can handle the items that are not appeared in the example set

The LOE task as Regresstion

The LOE task can be veiwed as a regression task targeting the orders

Obserbed Order (example) =

Absolute Order (model) + noise

Absolute Order: the order of all possible items

Noise: random swapping of items

The rule for sorting that is acquired by LOE tasks

= The description of the absolute order by the attributes of items

Evaluation method

Error measure:

Spearman's Rank Correlation (ρ coefficient)

The correlation between ranks of items in the two orders of the same item set

0: no correlation, 1: complete match, -1: reverse

t follows the Stuent *t*-distribution

with degree of freedum (#I–2)

$$t = \rho \sqrt{\frac{\# I - 2}{1 - \rho^2}}$$

$\# I$: the length of orders

Related Works (1)

Rank Correlation: Spearman's ρ or Kendall's τ

measure for comparing two orders

Paired Comparison:

Thurstone's method or Bradley's method

input: pairwise precedence information

= which precedes the orther between two items

output: real valued scale compatible with inputs as possible

The aim is different from LOE task:

not estimating the orders

Related Works (2)

[Cohen et al. 99]

input: pairwise precedence information

= which precedes the other between two items

output: estimate the order that preserves input information as possible

Difference from LOE task

target: **LOE:** totally well sorted orders

Cohen: preserving pairwise information

error evaluation:

LOE: final orders are directly evaluated by Spearman's ρ

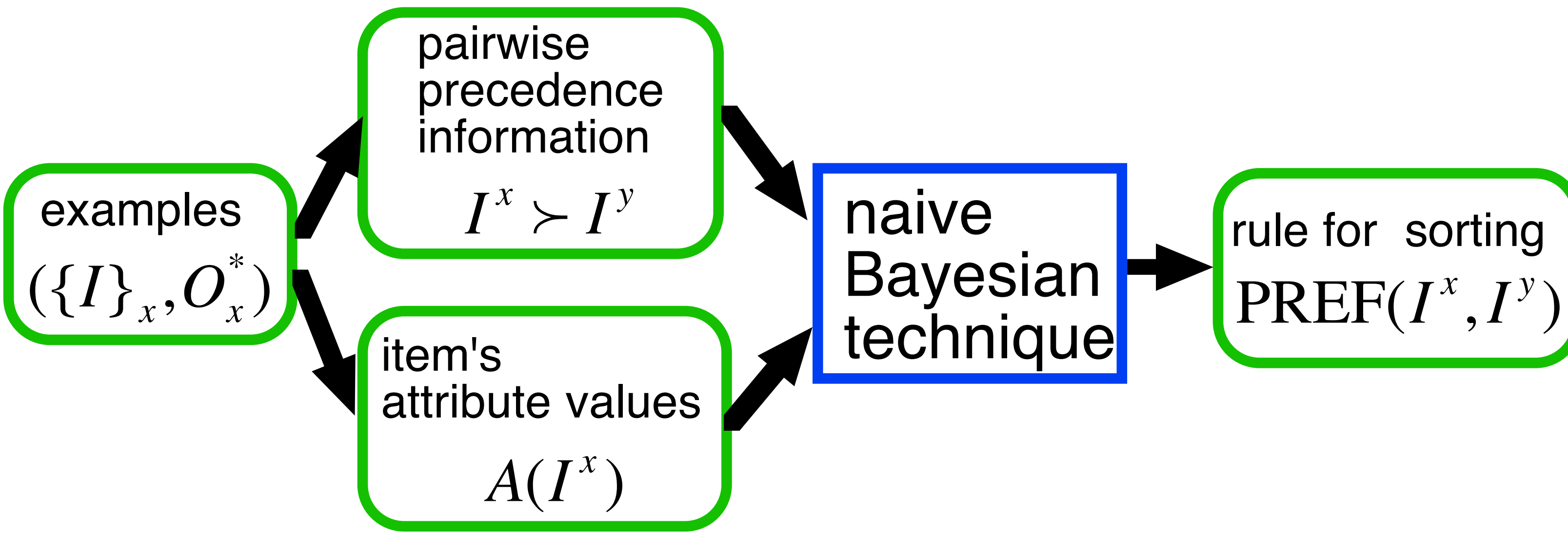
Cohen: evaluate the final orders indirectrly by the the accuracy of intermediate function

Method: Classification-Based (1)

Learning Stage:

rule for sorting = $\mathrm{PREF}(I^x, I^y)$

The conditional probability of the event $I^x > I^y$ given attribute values of these two items



Method: Classification-Based (2)

Sorting Stage:

estimated order for the unordered item set O_U

= the order maximizing the criterion function

- Find the order by greedy search techniques

Two types of functions were examined

type SC: $$\sum_{x,y:I^x \succ I^y} \mathrm{PREF}(I^x, I^y)$$

compatible with the Cohen's criterion

type PC: $$\prod_{x,y:I^x \succ I^y} \mathrm{PREF}(I^x, I^y)$$

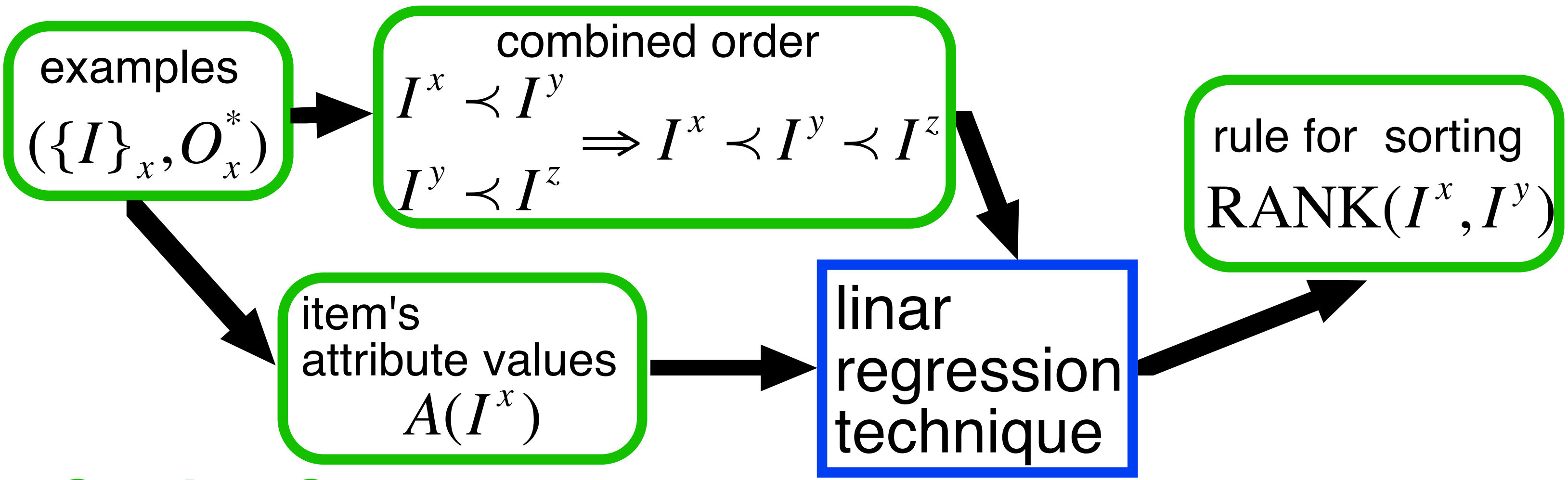
theoretical advantage over the Cohen's criterion

Method: Regression-Based

Learning Stage:

rule for sorting = $\mathrm{RANK}(I^x)$

function of item's attribute values that outputs the expected item's rank



Sorting Stage:

Find the order sorted according to RANK values

Artificial Data: Configuration

Aim of Experiments on Artificial Data:
To analyze the charactristics of these methods

- Item set types
  the number of attributes = {3, 4, 5}
  the number of values per one attribute = {3, 5, 7}
- Method to generate example orders
  10 orders are generated for each item set types
  orders are defined according to the score that is a linear function of weight or attribute values
- Example sets
  the length of example order (= the size of item sets) = {3, 5, 10}
  the number of examples = {10, 30, 50}
- Apply leave-one-out test (strict cross-validation)
- Error measure is the mean of ρ between the estimated order and the original order

Artificial Data: means of ρ

The means of ρ

	ALL	$\#I = 3$	$\#I = 5$	$\#I = 10$
SC	0.808	0.667	0.825	0.932
PC	0.808	0.667	0.825	0.932
R	0.802	0.617	0.837	0.950

- $\#I$: the length of exampel orders = the size of items
- Rank Correlation ρ : 1=complete match, -1=reverse order
- SC : Classification-Based with Cohen compatible criterion
- PC : Classification-Based with the PREF product criterion
- R : Regression-Based method

Overall Results:

- $\#I$ becomes large ➡ performance improves
- The number of examples increase ➡ performance improves

Classfication vs Regression

Compare the classification-based method
and the regression-based method

paired *t*-test: the difference between ρ is statistically significant or not

	ALL	$\#I = 3$	$\#I = 5$	$\#I = 10$
SC-R	1.4430	4.4143	–2.2272	–8.5784
PC-R	1.4626	4.4254	–2.3547	–8.5023

Blue: Classification-based method is better
Red: Regression-based method is better

The length of example orders are:

	short	long
better method:	classification based	regression based

Transitivity Consistency (1)

Accuracy of intermediates

A: The accracy of PREF function of Classification Based Method

B: Correlation between ordinal and combined orders of Regression Based Method

	ALL	$\#I$: the size of item sets			$\#EX$: the number of examples		
		3	5	10	10	30	50
A	0.864	0.800	0.869	0.922	0.805	0.881	0.906
B	0.792	0.689	0.803	0.883	0.796	0.787	0.793

Sammary

$\#I$ increse ↗ ➡ **A:** increase ↗ **B:** drastically increase ⬆

$\#EX$ increse ↗ ➡ **A:** increase ↗ **B:** not change ➡

Transitivity Consistncy (2)

The performance of the regression-based is worse if the length of example orders is short. But, for longer lenth, it surpasses the classification-based. **WHY?**

This is results from the performance of intermediates. The better combined orders can be derived from the longer example orders. **WHY?**

The longer example orders **HIGHLY** preserve the transitivity consistency: $(I^x \prec I^y) \wedge (I^y \prec I^z) \Rightarrow I^x \prec I^y \prec I^z$

The Regression-Based method: This makes easy to combine of example orders

The Classification-Based method: This is not contribute the performance ⬅ independency assumption of pairwise precedence info is violated

Additional Experiment on SC

SC method (= compatible with Cohen's method)

Find the sub-optimal order by greedy search

Order derived by optimal search

➡ pairwise precedence info. will more preserved

	Optimal Search		Greedy Search	t-values of differnece
rank corerelation	0.805	<	0.808	−2.7915

the order estimated by optimal search is significantly worse than that by greedy search

Cohen's GOAL: order preseving pairwise precedence info

LOE's GOAL: totally well sorted orders

DIFFER!

Real Data: Overall

To investigate the LOE solution methods work well on Real Data

An Experiment:

Ask 52 people to sort 10 types of sushi according to his/her preference

By applying any of three methods,
we could acquire the order of which mean correlation to given preference orders is **moderately high**

➡ **Our LOE methods works well on real data**

Real Data: Further Analysis

The order derived by Regression-based method

like ... dislike

toro, hma-chi, tai, ikura, uni, ko-hada, tekka-roll, ika, salad-roll, kappa-roll

toro ← fatty tuna; kappa-roll ← cucumber

➡ One can know a summary of respodents in terms of preferences in sushi

ex. most popular type of sushi is "fatty tuna"

Rank correlation between the above order and the second author's preference order is HIGH (0.842)

➡ He has ordinal tendency of preference in sushi

Computational Complexity

	Learning Stage	Sorting Stage
PC SC	$\sum_{i}^{\#EX} (\#I_i)^2$	$(\#I_U)^3$
R	$(\#I_C)^3$	$\#I_U \log(\#I_U)$

$\#EX$: the number of examples
$\#I_i$: the size of i-th example item set
$\#I_C$: the size of combined item set
$\#I_U$: the size of the unordered item set

Learning Stage: classification-based method is btter

Sorting Stage: regression-based method is better

Conclusions

- We proposed a learning task that handles orders
- We showed several naive methods and analyzed these methods by applying them on artificial data
- We showed that these methods worked well also on real data

Errata

- In the last paragraph of the Section 6,

  (the first author's) ➡ (the second author's)